

An Experimental Design Approach for Optimizing SMSE Waveforms to Minimize Coexistent Interference

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Abstract—An experimental design approach is used to determine which factors (design parameters) of Spectrally Modulated, Spectrally Encoded (SMSE) waveforms have the greatest impact on coexistence with other communication waveforms. The SMSE framework supports cognition-based, software defined radio (SDR) applications and is well-suited for coexistence analysis. For initial proof-of-concept, a two factor (parameter), three-level (value) experimental design technique is applied to a coexistent scenario to characterize SMSE waveform impact on Direct Sequence Spread Spectrum (DSSS) receiver performance. The experimental design methodology reliably captures factor-level sensitivities and identifies those factors having greatest impact on system coexistence behavior (bit error variation). Given these initial results and its effectiveness in other engineering fields, it is believed that experimental design may pave the way for developing more rigorous waveform design methods and allow more robust coexistence analysis of conventional, DSSS and SMSE waveforms.

I. INTRODUCTION

As the demand for higher data rates (bandwidth) continues to increase, so does the challenge facing communications engineers who are seeking to design systems that can effectively coexist within common signaling domains (spatial, spectral, temporal, etc.). The goal is to maintain “tolerable” interference conditions for all users. Given the large number of potential parameter-value combinations, both within and across these domains, the design challenge requires a systematic approach for finding an effective solution without resorting to exhaustive searching and testing. Experimental design techniques provide a structured, organized method for determining how the relationship between factors (waveform parameters) and levels (parameter values), affect the process output (end-to-end system bit error rate) [1], [2]. Although the experimental design methodology has been extensively employed throughout the manufacturing industry, its application to the communication system design appear somewhat limited. However, recent success in multiuser detection applications [3] suggest that experimental design is a reasonable starting point for the coexistent SMSE scenario considered here.

As recently proposed [4], the analytic SMSE framework was established to embody a broad class of communication waveforms which are fundamentally based on Orthogonal Frequency Division Multiplexing (OFDM) principles, i.e., communication

symbol generation through spectral weighting of discrete frequency components (carriers) followed by inverse fast Fourier transformation (IFFT). The SMSE framework was developed to functionally incorporate six waveform design parameters that account for *data*, *coding*, *windowing*, *orthogonality*, *frequency assignment* and *frequency use*. These parameters are actually $1 \times N_f$ complex vectors where N_f is the number of components used in the IFFT process. Applicability of the SMSE framework has been demonstrated for several existing OFDM-based communication techniques using specific parameter realizations [5], [6], [7], [8]. However, the practical utility of the SMSE framework is believed to extend well-beyond its capability to readily implement existing waveforms with *given* parameter realizations. The functional incorporation of six waveform design parameters (factors), which can take on a broad range of values (levels), suggests the the SMSE framework is ideally suited for optimization through experimental design methods, i.e., determining *optimal* SMSE parameter realizations for a given output response variable(s).

Results presented here represent a first step toward demonstrating how the optimization process might work. For initial proof-of-concept, a two factor (parameter), three-level (value) experimental design technique is applied to a coexistent scenario to characterize SMSE waveform impact on DSSS receiver performance. The DSSS system bit error rate is used as the output response variable of interest. Experimental design results for two experiments (cases) are presented to illustrate that the method does indeed allow one to determine “optimal” parameter values (actually only “better” at this point since only two cases are presented).

II. SMSE ANALYTIC FRAMEWORK

In general, the SMSE model specifies the transmitted waveform design for the k^{th} data symbol using a specific collection of SMSE waveform design parameters, including: *coding*, $\mathbf{c} = [c_1, c_2, \dots, c_{N_f}]$, $c_i \in \mathbf{C}$, *data modulation*, $\mathbf{d} = [d_1, d_2, \dots, d_{N_f}]$, $d_i \in \mathbf{C}$, *windowing*, $\mathbf{w} = [w_1, w_2, \dots, w_{N_f}]$, $w_i \in \mathbf{C}$, and a phase-only *orthogonality* term, $\mathbf{o} = [o_1, o_2, \dots, o_{N_f}]$, $o_i \in \mathbf{C}$, $|o_i| = 1 \forall i$. Each of these terms are introduced to functionally incorporate

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various waveform design characteristics commonly employed in communications [4], [5], [6].

All that remains to completely specify the SMSE waveform is the frequency component selection and symbol duration of the resulting waveform. The frequency component defines the number of carrier components that are spectrally modulated and encoded. Assuming an N_f -point inverse fast Fourier transform (IFFT), there are initially N_f possible carrier components available. Use of components from this pool of frequencies is controlled through *assignment* and *used* parameters. For example, a system may elect to assign a subset of N_f carriers to a given user. This *assignment* is accounted for through $\mathbf{a} = [a_1, a_2, \dots, a_{N_f}]$, $a_i \in \{0, 1\}$, where zeros indicate unassigned carriers. From this assigned pool of carriers, some may go unused due to excessive interference, system design, etc. The remaining *used* carriers are accounted for through $\mathbf{u} = [u_1, u_2, \dots, u_{N_f}]$, $u_i \in \{0, 1\}$, where zeros indicate unused carriers and there are $P \leq N_f$ used frequencies. Clearly, \mathbf{u} is a subset of \mathbf{a} , $\mathbf{u} \subseteq \mathbf{a}$, and only assigned carriers are used.

Accounting for all SMSE design parameters, the framework provides a unified approach for generating and characterizing a host of OFDM-based signals. Using \odot to denote Hadamard product (element-by-element multiplication), the spectral representation of the k^{th} SMSE symbol is given by [4]

$$\mathbf{s}_k = \mathbf{a} \odot \mathbf{u} \odot \mathbf{c} \odot \mathbf{d}_k \odot \mathbf{w} \odot \mathbf{o}_k, \quad (1)$$

where the m^{th} carrier component of \mathbf{s}_k is given by

$$s_k[m] = a_m u_m c_m d_{m,k} w_m e^{j(\theta_{d_{m,k}} + \theta_{c_m} + \theta_{w_m} + \theta_{o_{m,k}})} \quad (2)$$

for $m = 0, 1, \dots, N_F - 1$ frequency component indices with $c_m, \theta_{c_m}, d_{m,k}, \theta_{d_{m,k}}, w_m, \theta_{w_m}$ and $\theta_{o_{m,k}}$ being the corresponding magnitudes and phases of the design parameters. By design, the coding and windowing terms only vary with frequency index m whereas the data modulation and orthogonality terms vary with symbol index k as well.

III. EXPERIMENTAL DESIGN

Real-world design problems often depend upon the successful manipulation of several input factors (parameters) which may take on several valid levels (values). Generally, the factor-level combinations have competing effects on desired output response variables. As introduced earlier, the coexistent communication system design categorically fits within this class of problems. In the case of continuous parameters, the number of allowable levels is actually infinite. Thus, factors are often tested one-at-a-time, a method that ignores the interactions between factors. One set of techniques, collectively known as *Experimental Design*, has proven itself in industrial experimentation and typically involves quality control. However, it is believed that this same approach may be useful in other types of engineering applications, including waveform design. Given experimental design has a solid foundation in linear systems theory, it is readily accessible for communications design [1].

A. Factorial Experiments

A factorial experimental design contains every combination of factors (parameters) and their corresponding levels (values). Therefore, to explore the impact of N factors on a given output response variable with each factor taking on M different levels, the full factorial design would require an integer multiple of M^N runs. Such a design allows the investigation of both the main effects and their interactions on the response. In a one-at-a-time design, only the main effects are considered. Even when only the main effects are of interest, a factorial design often provides equal quality estimates of the main effects with fewer experimental runs. In other words, a factorial design is more efficient in terms of experimental runs and also provides the ability to more accurately model a system by including factor interactions [1].

For convenience, the experiment is described in terms of *coded variables*, which are a mapping of natural variable units to simple, often symmetrical values, such as -1, 0, and 1 [1]. For example, if a two-factor, three-level full factorial experimental design was desired, the experiment would require a multiple of $n = 3^2$ runs, representing every combination of two sets of -1, 0, and 1. In addition, *center runs*, or more runs of the $\{0, 0\}$ combination, are often added to the experiment for better error estimates [1].

B. Analysis of Variance

Analysis of Variance (ANOVA) is at the heart of *Design of Experiments*. It is a statistical technique which uses the sample variances of a data set to test the impact of input parameters on an output response variable. As a result, ANOVA provides insight into the shape of the response surface by including appropriate while eliminating inappropriate model terms. The ANOVA process begins by assuming a model for the test data [2]. For this experiment the assumed model is *second-order* and expressed as [1]

$$y_{ijk} = \beta_0 + \beta_1 x_{1i} + \beta_2 x_{2j} + \beta_{12} x_{1i} x_{2j} + \beta_{11} x_{1i}^2 + \beta_{22} x_{2j}^2 + e_{ijk}. \quad (3)$$

In the above equation, y is the response variable to the coded input variables, x_1 and x_2 . The β terms represent the regression coefficients and e represents the error. The suffixes i and j represent the index values of x_1 and x_2 respectively, while the suffix k represents the run number. This expression may be expressed more compactly in matrix form, as shown in (4).

$$\mathbf{Y} = \mathbf{X}\beta + \mathbf{e} \quad (4)$$

Essentially, the ANOVA process performs a least-squares fit of the data to the model by applying (5) [1], [2].

$$\hat{\beta} = (\mathbf{X}^T \mathbf{X})^{-1} \mathbf{X}^T \mathbf{Y} \quad (5)$$

Substituting this estimate for β into (4) yields the regression model, (6).

$$\hat{\mathbf{Y}} = \mathbf{X}\hat{\beta} = \mathbf{X}(\mathbf{X}^T \mathbf{X})^{-1} \mathbf{X}^T \mathbf{Y}, \quad (6)$$

Finally, the error is calculated by (7).

$$\mathbf{e} = \mathbf{Y} - \hat{\mathbf{Y}} \quad (7)$$

After the least-squares, the sample variance is partitioned into subspaces corresponding to the main effects, interactions, and error. The variances of the main effects and interactions are compared to the error variance as a significance test for their relative importance to the model. More specifically, assuming that the underlying model error is Gaussian, the ratio of a factor's sample variance to the error's sample variance will be distributed according to the F distribution. Tabulated values of the F distribution may then be used to provide probabilities that the factor under test is significant, that is, it should be included in the model. Therefore, ANOVA provides a tool to decide which factors and interactions are important in a system model, a task which is often left to intuition [1], [2]. For more information regarding ANOVA calculations see [1].

C. Response Surface Methodology

Because the ANOVA performs a least-squares data fit to the assumed model and provides information about the appropriate model terms, it is relatively straight forward to use its results to find a *stationary point*, which may be a maximum, minimum, or saddle point [1]. If ANOVA analysis shows that the surface is quadratic, such that the x_1^2 and x_2^2 terms are significant, then the stationary point may be determined by (8). A maximum or minimum indicates an optimal combination(s) of input factors.

$$\mathbf{x}_s = -\frac{1}{2}\mathbf{B}^{-1}\mathbf{b} \quad (8)$$

In the two-factor case, the matrices are defined in the following manner [1].

$$\mathbf{b} = \begin{bmatrix} \hat{\beta}_1 \\ \hat{\beta}_2 \end{bmatrix}, \quad \mathbf{B} = \begin{bmatrix} \hat{\beta}_{11} & \frac{\hat{\beta}_{12}}{2} \\ \frac{\hat{\beta}_{21}}{2} & \hat{\beta}_{22} \end{bmatrix}$$

After solving for the stationary point, the eigenvalues of \mathbf{B} indicate its type. If both of the eigenvalues are negative, the two input variables create a maximum in the response surface. If both of them are positive, the input variables create a minimum. If the signs of the eigenvalues differ, it is a saddle point [1].

If a stationary point is not present, then the response surface may be searched by gradient methods, such as the *method of steepest descent* [1].

Therefore, the following algorithm provides a simple, efficient methodology.

- 1) Perform a three-level full-factorial experiment upon a small region of interest within the larger parameter space.
- 2) Use the ANOVA data analysis technique to determine the appropriate terms to be included in the second order model.
- 3) If the ANOVA suggests that the quadratic terms are not significant, then search the gradient via.
- 4) If the ANOVA suggests the quadratic terms are highly significant, then solve for the stationary point via (8).

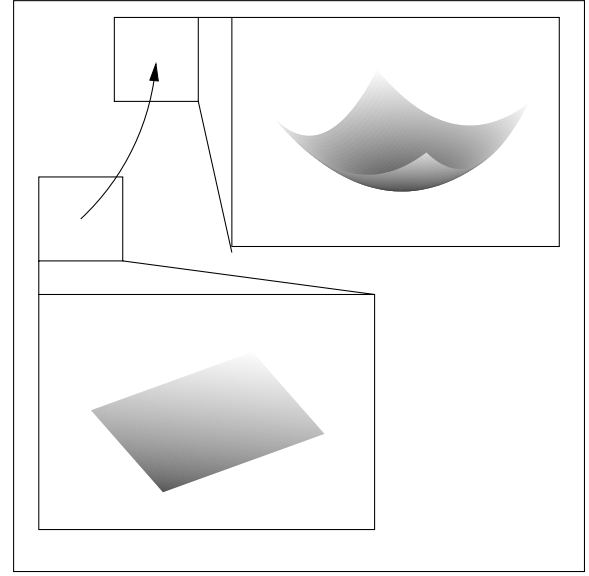


Fig. 1. This figure illustrates the concept of using a gradient search to find a quadratic region in the response surface. The ANOVA for the initial region suggested that the response surface was linear. After following the gradient using the method of steepest descent, indicated by the arrow, a quadratic region was found. In this example, the region contains a stationary point which is a local minimum.

- 5) When a stationary point is found, verify its type by eigenvalue analysis.

The process of searching via the steepest descent from a linear to a quadratic region is illustrated in Figure 1.

D. Significance Testing

Upon completing the experiments, traditional hypothesis testing, such as the t-test, may be used to confirm that results are indeed statistically significant. Paired comparisons test the significance between the output response variables from two different cases, while a single comparison test may be used to compare a output response variable with a fixed value, such as an ideal or limiting case. Also, confidence intervals are readily available for each outcome [1].

IV. COEXISTENCE APPLICATION

1) *Demonstration Scenario:* To demonstrate the experimental waveform design procedure, a coexistence scenario is considered whereby the performance of a DSSS receiver is evaluated in a coexistent environment containing an SMSE waveform. Ultimately, the experimental design goal would involve finding the SMSE factor-level combination(s) that have minimal impact on DSSS receiver performance, i.e., those inducing least degradation on DSSS bit error performance.

2) *DSSS System:* Direct sequence spread spectrum (DSSS) is commonly used in modern communication systems, e.g., the digital cellular IS-95 system and the Global Positioning System (GPS) are both DSSS systems. By re-modulating the data

TABLE I

THE VALUES FOR THE INITIAL EXPERIMENT ARE SHOWN IN THE TABLE. THE FACTORS ARE THE NUMBER OF SMSE CARRIERS, N_f AND THE SYMBOL DURATION, T_s . THE x_1 AND x_2 COLUMNS REPRESENT THE CORRESPONDING CODED VALUES FOR THE TRIAL.

Factor Level	N_f	x_1	T_s	x_2
Low	2^3	-1	58.8	-1
Medium	2^4	0	62.5	0
High	2^5	1	66.7	1

modulated waveform with a spreading waveform, the DSSS system generates a signal having 1) a bandwidth that is much greater than the original data modulated waveform, and 2) a reduced power spectral density response. Binary phase-shift keying (BPSK) is commonly used for the spreading modulation. Upon reception, the DSSS waveform is despread using the original spreading waveform. Ideally, this returns the signal back to its original form prior to data demodulation [9], [10].

The spreading and despreading process of the DSSS system inherently provides several benefits. First, by assigning unique spreading codes to network users in what is called code division multiple access (CDMA), multiple users can share a common spectral band. Second, the reduced power spectral density enhances coexistence with narrowband systems (minimizes interference users outside the network) given that less DSSS power exists within their bandwidth of operation. Finally, the despreading operation provides some suppression of coexistent interfering signals, whether they are from other communications systems or multipath [11], [9].

The DSSS system considered here uses BPSK for both data and spreading modulations. The spreading code is a 32-bit Hadamard sequence with exactly one code period occurring per data symbol. The model also assumes an Additive White Gaussian Noise (AWGN) channel. The DSSS receiver is perfectly synchronized to the transmitted wave and the RF front-end filter is modeled as being ideal. Communication symbols are estimated using a single channel correlation receiver.

3) *SMSE System*: The SMSE coexistent signal is generated using the framework of (2) with all but two design parameters from Section II fixed to implement conventional OFDM [12]. As shown in Table I, two SMSE factors (design parameters) are varied for the experiment: the total number of carriers which dictates the number of IFFT points and the symbol duration of the data modulated carriers. Together, the total number of carriers and symbol duration also dictate the overall waveform bandwidth.

4) *Experimental Design*: The experiments assume the second-order model from (3). As a result, the experimental design consists of a two-factor, three-level, full-factorial design with four additional center runs. The matrix form of the system model from (4) is then expressed as (9).

TABLE II

THE VALUES FOR THE FINAL EXPERIMENT ARE SHOWN IN THE TABLE. THE FACTORS ARE THE NUMBER OF SMSE CARRIERS, N_f AND THE SYMBOL DURATION, T_s . THE x_1 AND x_2 COLUMNS REPRESENT THE CORRESPONDING CODED VALUES FOR THE TRIAL. THE MAPPING WAS DETERMINED BY FOLLOWING THE LINE OF STEEPEST DESCENT, BEGINNING WITH THE VALUES IN TABLE I.

Factor Level	N_f	x_1	T_s	x_2
Low	2^1	-1	61.1	-1
Medium	2^2	0	63.0	0
High	2^3	1	65.1	1

$$\mathbf{Y} = \begin{bmatrix} 1 & -1 & -1 & 1 & 1 & 1 \\ 1 & -1 & 0 & 0 & 1 & 0 \\ 1 & -1 & 1 & -1 & 1 & 1 \\ 1 & 0 & -1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 & 1 \\ 1 & 1 & -1 & -1 & 1 & 1 \\ 1 & 1 & 0 & 0 & 1 & 0 \\ 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \beta_0 \\ \beta_1 \\ \beta_2 \\ \beta_{12} \\ \beta_{11} \\ \beta_{22} \end{bmatrix} \quad (9)$$

The mappings between the natural units and the coded unit in the \mathbf{X} matrix of (4) are shown in Tables I and II. The first table shows the mapping for the initial trial, while the second table shows the final trials mapping after applying the method of steepest descent.

V. EXPERIMENTAL DESIGN RESULTS

Tables III contains the ANOVA tables for the initial and final experiments. The different rows of the table represent the main effects and interactions according to the model of the experiment. As previously stated, the F distribution provides the significance test for each factor. The larger the number in the F column, the more likely that term is significant. In these trials, $F_0 = F_{0.05,1,7} = 5.59$ was the significance threshold.

As one may observe, the results of the first trial indicated that the response surface was not quadratic, because the value of the F statistic for the x_2^2 term is not greater than 5.59. After moving to the final region, however, both quadratic terms are significant, and the stationary point is found by (8) to be $N_f = 2$ and $T_s = 63.0$ msec.

Eigenvalue analysis of the resulting regression coefficients determines, however, that the stationary point is neither a minimum or a maximum, but instead a saddle point. Therefore the search must be repeated using a different starting point.

To illustrate consistency with physical waveform level modeling, an end-to-end simulation was run for the SMSE-DSSS

TABLE III

ANOVA TABLE FOR THE INITIAL AND FINAL TRIAL. NOTE THAT THE F STATISTIC FOR THE x_2^2 TERM DOES NOT EXCEED THE THRESHOLD OF 5.59 IN THE FIRST TRIAL, BUT BOTH QUARATIC TERMS DO EXCEED IT FOR THE FINAL TRIAL. AS A RESULT, (8) MAY BE USED TO SOLVE FOR THE LOCATION OF THE STATIONARY POINT AFTER THE FINAL TRIAL.

Source of Variation	Degrees of Freedom	Initial F	Final F
Model	5	168.06	49.35
x_1	1	820.61	124.34
x_2	1	3.61	2.77
x_1x_2	1	1.18	8.14
x_1^2	1	14.57	110.97
x_2^2	1	0.86	11.25
Error	7		

coexistent scenario. The results are shown in Figure 2. They include the bit error curve for the stationary point found in the example, as well as the best and worst case scenarios from multiple searches.

VI. CONCLUSION

Preliminary results show promise for using an experimental design approach for designing SMSE waveforms that minimize mutual interference in coexistent scenarios. The proof-of-concept results presented herein suggest that experimental design techniques may pave the way for more rigorous co-existence analysis of conventional, DSSS and OFDM-based SMSE waveforms. For initial proof-of-concept, a two factor (parameter), three-level (value) experimental design technique

was applied to a coexistent scenario to characterize SMSE waveform impact on DSSS receiver performance. Considering DSSS system bit error rate as the output response variable, the experimental design results were consistent in predicting system bit error behavior, i.e., for various changes in SMSE waveform design levels the output bit error rate responded as expected. Research is currently underway to greatly expand the preliminary results by:

- Using an output response variable Y that *simultaneously* accounts for the probability of bit error performance for both systems being considered.
- Using the expanded SMSE framework shown in (2) to allow the testing of many types of waveforms, some of which may be new.

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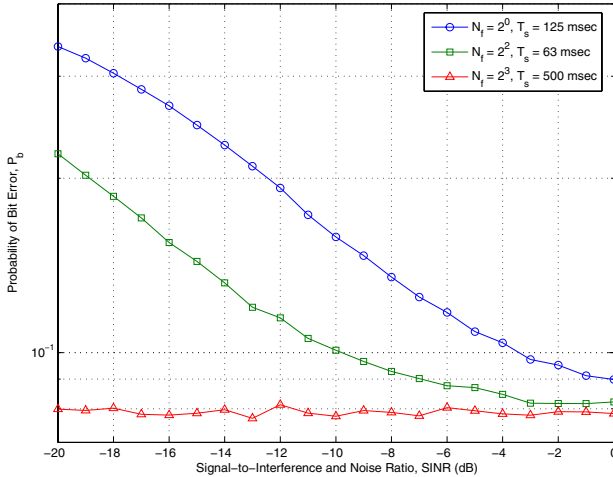


Fig. 2. Bit error rate (P_b) versus DSSS Signal Power-to-SMSE Interfering Power (S/I) ratio for each case. The initial and final parameters, ($N_f = 2^4, T_s = 200$ msec) and ($N_f = 2^1, T_s = 63$ msec), demonstrate that the local minimum does provide marginal improvement over the starting point. The two extreme cases of ($N_f = 2^3, T_s = 500$ msec) and ($N_f = 2^0, T_s = 125$ msec) represent the best and worst cases found over multiple searches, respectively.